

Register No: _____

THE KAVERY ENGINEERING COLLEGE

(An Autonomous Institution)

B.E/B.TECH DEGREE END SEMESTER THEORY EXAMINATIONS, NOV /DEC 2024

Third Semester

Electrical and Electronics Engineering

MA3303 - Probability and Complex Functions

Regulations - 2021

Duration : 3 Hrs

Maximum : 100 Marks

PART - A (ANSWER ALL QUESTIONS) (Marks 10*2=20)

<i>Q.No</i>	<i>Questions</i>	<i>BLT</i>	<i>CO</i>	<i>Marks</i>
1.	A pair of dice is tossed twice. Find the probability of scoring 5 points at least once.	UN	1	2
2.	Check whether the function $f(x) = \frac{x-2}{2}$ for $x=1, 2, 3, 4$. Can serve as probability distribution.	UN	1	2
3.	The life time of a certain brand of an electric bulb may be considered a random variable with mean 1200 h and standard deviation 20 h. Find the probability, using central limit theorem, that the average lifetime of 10 bulbs exceeds 1250 h.	UN	2	2
4.	Define covariance of random variable X and Y. Define their independence using covariance.	RE	2	2
5.	Test the analyticity of the function $w = \sin z$.	UN	3	2
6.	What is conformal mapping?	RE	3	2
7.	Evaluate $\int_c \frac{e^z}{z(1-z)^3} dz$ if 0 lies inside c and 1 lies outside c.	UN	4	2
8.	Find the nature of the singularity of $f(z) = \frac{z - \sin z}{z^2}$.	UN	4	2
9.	Solve the differential equation $(D^2+5D+6)y=0$.	UN	5	2
10.	Build the equation $x^2y''+xy'=x$ into linear differential equation with constants coefficients.	UN	5	2

PART – B (ANSWER ALL QUESTIONS) (Marks 5*16 = 80)

Q.No	Questions		BLT	CO	Marks														
11.	a i	A bag contains 7 balls and is not known how many of them are white. Two balls are drawn at random from the bag and they are noted to be white. What is the chance that all the balls in the bag are white?	AP	1	8														
	ii	Find the probability density function of $Y=8X^3$, given the random variable X with the density function $f(x)=\begin{cases} 2x, & 0 < x < 1 \\ 0, & otherwise \end{cases}$.	AP	1	8														
Or																			
	b i	Out of 800 families with 4 children each, how many families would be expected to have. (1) 3 boys and 1 girl (2) Atleast 1 girl	AP	1	8														
	ii	The time (in hour) to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$. (1) What is the probability that the repair time exceeds 3h? (2) What is the conditional probability that a repair takes at least 12 hours given that its duration exceeds 9 hours?	AP	1	8														
12.	a	Determine the value of c that makes the function $f(x, y) = c(x+y)$ a joint probability mass function over the nine points with $X=1, 2, 3$ and $Y=1, 2, 3$. Determine the following: (1) $P(X=5, Y<4)$ (2) $P(X=1)$ (3) $P(Y=4)$ (4) Marginal probability distribution of the random variable X (5) Find correlation and covariance	AP	2	16														
Or																			
	b i	Determine the value of c such that the function $f(x,y)=cxy$, for $0<x<3$ and $0<y<3$ satisfies the properties of a joint probability density function. Determine the following: (1) $P(X<2, Y<3)$ (2) $E(X), E(Y)$ (5) Conditional probability distribution of Y given that $X=1.5$.	AP	2	8														
	ii	Find a least square straight line for the following data																	
		<table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>Y</td><td>6</td><td>4</td><td>3</td><td>5</td><td>4</td><td>2</td></tr></table>	X	1	2	3	4	5	6	Y	6	4	3	5	4	2	AP	2	8
X	1	2	3	4	5	6													
Y	6	4	3	5	4	2													
13.	a i	If $f(z)$ is a regular function of z , prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) f(z) ^2 = 4 f'(z) ^2$.	AN	3	8														
	ii	If $f(z)=u+iv$, is analytic find $f(z)$ and v if $u=\frac{\sin 2x}{\cos 2x + \cosh 2y}$.	AN	3	8														
Or																			

	b i	Find the bilinear map which maps the points $z=1, i, 0$ onto $w=1, i, -1$.	AN	3	8
	ii	Show that the map $w = \frac{1}{z}$ maps the totality of circles and lines as circles or lines.	AN	3	8
14.	a	State and prove Cauchy's integral formula. Use this formula to evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where C is the circle $ z =3$.	AP	4	16
Or					
	b i	Find the Laurent's series expansion of $f(z) = \frac{7z-2}{z(z+1)(z-2)}$ in the region $1 < z < 3$.	AP	4	8
	ii	Evaluate the real integral $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$ by using Cauchy's residue theorem.	AP	4	8
15.	a i	Solve the equation $(D^3 - D^2 - 6D)y = x^2 + 1$.	AP	5	8
	ii	Solve $(D^2 + a^2)y = \tan ax$ by method of variation of parameters.	AP	5	8
Or					
	b i	Solve the equation $(x+2)^2 \frac{d^2 y}{dx^2} - (x+2) \frac{dy}{dx} + y = 3x + 4$.	AP	5	8
	ii	Solve the simultaneous equation $\frac{dx}{dt} + y = e^t, x - \frac{dy}{dt} = t$.	AP	5	8